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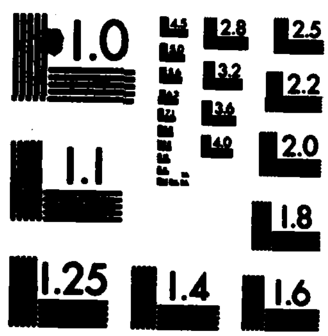

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ANALYSES OF ENERGY EXTRACTION
EFFICIENCY OF UNSTABLE RESONATORS

J206-84-005/6239

Final Report

by

Jui-teng Lin

March 30, 1984

Prepared for

Naval Research Laboratory
4555 Overlook Avenue, SW
Washington, DC 20375

Under:

Contract Number N00014-83-C-2314

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March 30, 1984

Dr. John F. Reintjes
Code 6542
Naval Research Laboratory
4555 Overlook Avenue, SW
Washington, DC 20375

SUBJECT: Final Report, Contract Number N00014-83-C-2314

Dear Dr. Reintjes:

JAYCOR is pleased to submit this Final Report entitled *Analyses of Energy Extraction Efficiency of Unstable Resonators*, in accordance with the subject contract, CDRL Item Number A003.

If the Final Report is acceptable, please sign and forward the enclosed DD Form 250.

Questions of a technical nature should be addressed to Dr. Jui-teng Lin while questions of a contractual nature should be addressed to Mr. Floyd C. Stilley, our Contracts Administrator.

Sincerely,

Martin C. Nielsen

Martin C. Nielsen
Vice President and Counsel

ssh-b

Enclosures

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Positive branch, confocal, unstable resonator, outcoupling factor, efficiency, Rigrod analysis, optimalized magnification, rate equation, Newton's method, gain and loss ratio, mean-field approximation, far-field intensity.

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Energy extraction efficiency of a telescopic resonator is studied both analytically and numerically. It is shown that the usual Rigrod-type calculations are not valid for systems with curved mirrors. A generalized Rigrod analysis is presented and compared with the numerical results. Experimental measurements related to NRL's laser systems are proposed based on the present modeling.

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system. Ananev, et al., have recently developed numerical analysis on the efficiency of a telescopic resonator[3,4]. However, no tractable results are available for the achievement of the resonator design due to the nonlinear behavior of the rate equation and the coupled optical elements of the system.

In the present research report, we will analyze the optimum condition for the energy extraction from confocal, positive branch, unstable resonators for Excimer lasers. Both cylindrical and spherical mirrors for 2D and 3D gain elements, respectively, will be investigated. For tractable results, analysis on simplified systems will be investigated and compared with the exact numerical results for more realistic systems. The geometric-optics approximation, which has been shown to be valid for systems with large Fresnel numbers[5,6], will be employed, i.e., the small deviation of the eigenvalue due to the multipass diffraction loss and the non-uniform phase profile on the output mirror (within ten degrees for Fresnel number > 50) will be ignored.

The transverse and horizontal variation of the forward and backward intensity will be numerically calculated for the case of constant unsaturated gain. Analytic results based on the mean-field approximation will be derived and compared with the numerical results. The beam quality based on the focusability of the far-field will be analyzed. Finally, some suggested measurements which provide system parameters information such as optimal magnification and maximum output power and the gain/loss are discussed.

There are five appendices in this report including the computer code.

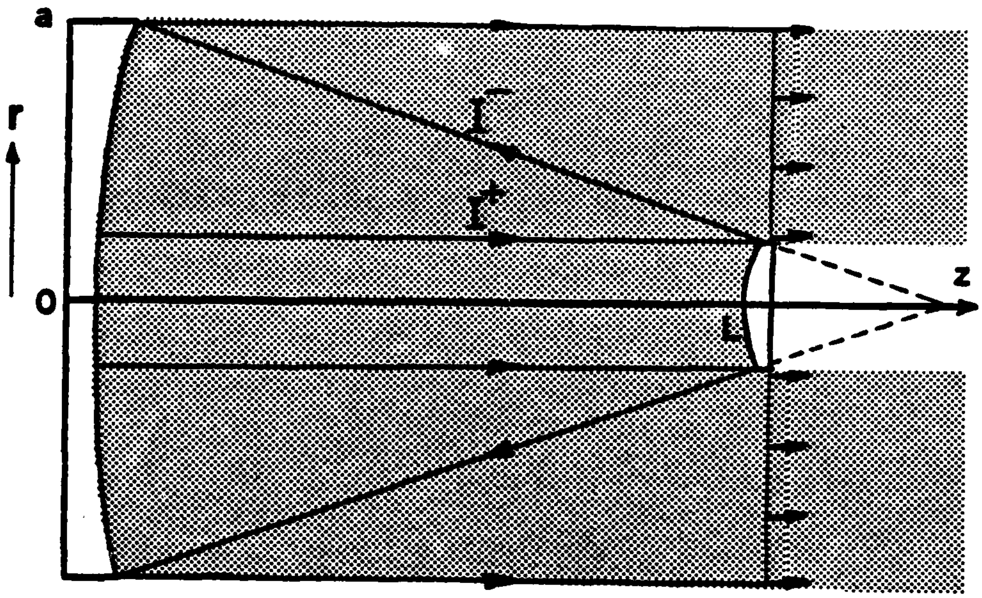
II. THE SYSTEM AND THE PROBLEMS

To obtain the energy extraction from a confocal unstable resonator, as shown in Figure 1, the intensity inside the loaded cavity whose gain function is usually given by a homogeneous broadening should be calculated. Subjected to the oscillation threshold condition (loss = gain), the maximum energy extraction may be obtained by optimizing system parameters such as reflectivity and magnification. As shown by the flowchart in Figure 2, our final goal is to find the resonator efficiency defined by [output power]/[gain], where the output power is given by

$$P = \frac{1}{S} \int_{a/M}^{a} I_{out} dS'. \qquad [1]$$

Here I_{out} is the output intensity evaluated at $z = L$ and is integrated over the outcoupling regime and normalized by S; $dS' = 2\pi r dr$, $S = \pi a^2$ for spherical mirrors and $dS' = dr$, $S = a$ for cylindrical mirrors.

In general, the output intensity cannot be solved analytically from the rate equation and hence computer simulation is needed for the resonator efficiency. We note that the conventional analysis based on $P = (1 - M^{-n}) I_{out}$ oversimplifies the system and the assumption of a constant I_{out} may cause considerable error in the prediction of the energy extraction. Before showing our new technique for obtaining resonator efficiency without assuming a constant I_{out}, we will address some of the features of a confocal resonator which are significantly different from those of the Fabry-



Reflectivities:	R_1, R_2
Magnification:	M
Gain cavity length:	L
Gain function:	g (r, z)
Loss function:	α
Effective reflectivity:	$Reff = R_1 R_2 M^{-n}$
Dimensionality:	n = 1 (2D), n = 2 (3D)
Common focus	$Z_o = ML/(M - 1)$
Threshold condition:	$Reff \exp [\int_0^{2L} (g - \alpha)\, dz] = 1$ (F.1)

Rate equations: $$\frac{1}{I^+} \frac{\partial I^+}{\partial z} = (g - \alpha)$$

$$r \frac{\partial I^-}{\partial r} - (Z_o - z) \frac{\partial I^-}{\partial z} = [(g + \alpha)(Z_o - z) - n] I^- \quad \text{(F.2)}$$

Figure 1 Schematic diagram of a confocal resonator and key system equations.

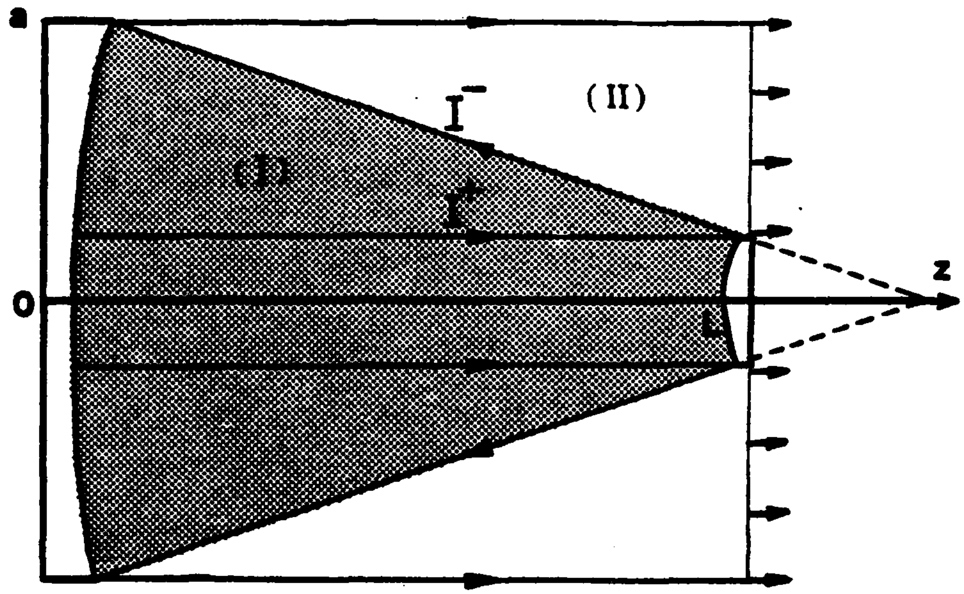


Figure 2. Schematic diagram for a confocal resonator for an Excimer laser and the associated intensity mesh-point map in the bi-directional (I) and pure amplification (II) regime.

Perot resonators and the factors which cause the complexity in our system, as follows:

(i) The spatial non-uniformity of the gain function.

(ii) The mirror curvatures which guide the propagation of the forward (I^+) and backward (I^-) intensity differently.

(iii) The r-dependence of the threshold condition.

(iv) The z-dependence of the total intensity ($I^+ + I^-$) and their product I^+I^-.

(v) The variation of the output intensity in the transverse coordinate.

We note that in the Rigrod analysis for a Fabry-Perot resonator (with M = 1) most of the above described features are ruled out, and hence simple expression for the efficiency is available. Our numerical results of the rate equation will show features (iv) and (v). It is seen that the spatial variations of the intensity $I^{\pm}(z)$ and $I_{out}(r)$, which are the essential feature for the confocal resonators, cannot be ignored, e.g., Equation [1] should be integrated rather than being multiplied by a constant outcoupling fraction $(1-M^{-n})$.

III. ANALYTIC TREATMENT

For tractable results, we shall first investigate an analytical expression for the resonator efficiency η. Assuming a constant mean-intensity in the gain function, the on-axis threshold condition gives a simple expression for the resonator efficiency (see Appendix A)

$$\eta_0 = \frac{X}{g_0 L}\left[\frac{g_0 L}{\alpha_0 L + X} - 1\right], \qquad [2]$$

$$X = 1/2 \ln R_{eff}^{-1}, \qquad [3]$$

where we have introduced an effective reflectivity $R_{eff} = R_2 M^{-n}$, R_2 being the reflectivity of the output mirror (for $R_1 = 1$) and M the magnification of the confocal resonator with dimension parameter $n = 1$ for 2D (cylindrical) and $n = 2$ for 3D (spherical) mirrors. The important feature of the above generalized expression is twofold:

- For a Fabry-Perot system, $M = 1$ Equation [2] reduces to the Rigrod results; and
- For a confocal resonator with $M > 1$, Equation [2] provides a new expression which fits the numerical results better than those of conventional expression with a coupling fraction $(1 - M^{-n})$.

The maximum efficiency (η^*) may be calculated by the optimizing condition $d\eta/dX = 0$,

$$\eta_0^* = (1 - \frac{1}{\sqrt{D}})^2, \qquad [4]$$

with the associated optimal value

$$X_{opt} = g_0L\ (\frac{1}{\sqrt{D}} - \frac{1}{D}), \qquad [5]$$

where $D = (g_0L/\alpha_0L)$ is the unsaturated gain and loss ratio. Some of the salient features of the above analytic expressions are:

(i) As D increases, the maximum efficiency increases and reaches unit when D approaches infinite;

(ii) In a given reflectivity, say $R_1 = R_2 = 1$, the optimal magnification is, from Equation [5], given by

$$M^n_{opt} = \exp\ [g_0L\ (\frac{1}{\sqrt{D}} - \frac{1}{D})], \qquad [6]$$

which is shifted to the larger value when the total gain of the system increases for a given internal loss. We should note that the validity of Equation [4] is limited by the inequality[3]

$$f_1 = (1 - \frac{1}{\sqrt{D}})^2 < [1 - \exp\ (-\ \alpha_0L)]/\alpha_0L = f_2, \qquad [7]$$

Some of the examples are: for $D = 10$, $f_1 = 0.467$, we have $f_2 = 0.432$ when $\alpha_0L = 2$; for $D = 20$, $f_1 = 0.6$, we get $f_2 = 0.632$ when $\alpha_0L = 1$. Therefore the maximum energy extraction

of an unstable resonator is limited by the cavity length which is in turn limited by the inequality, Equation [7]. For systems with long active media, the maximum efficiency is given by the smaller quantity, either f_1 or f_2. We shall compare the aforementioned features with our numerical results later.

IV. NUMERICAL RESULTS

The rate equations, (F.2) in Figure 1, subjected to the steady-state (on-axis) condition will be solved numerically. Due to the curvatures of the mirror, the forward and backward fields are propagating differently. We shall investigate the intensity profiles across the output mirror and the propagation of the forward and backward field. It will be shown that the transverse variation of the intensity across the output mirror is mainly caused by the pure amplication of the forward intensity when it enters regime (II), as shown in Figure 2. In the following discussion, we shall consider the situation where $|dI_-/dr| << |dI_-/dz|$ and assume a constant unsaturated gain, i.e., g_0 = constant. However the total gain will be strongly z-dependent due to the z-dependent total intensity.

In order to employ Newton's method, we require an explicit form of the total intensity, i.e., the z-dependence of the gain function must be known. As shown in Appendix B, in contrast to the planar-mirror system, $I_+(z)$ and $I_-(z)$ and their product are strongly dependent on z due to the expansion of the backward field, $I_-(z)$, and the accompanying gain saturation which in turn affects the z-dependence of the forward intensity, $I_+(z)$. The numerical procedures are shown in Figure 3 with greater detail shown in Appendix B. The initial intensities, $I_\pm(z = 0)$ are solved from the steady-state condition and the propagation of them gives us the on-axis intensity profiles. The numerical results are shown in Figure 4.a. The off-axis intensity of the forward field across the output mirror plane, z = L, is shown in

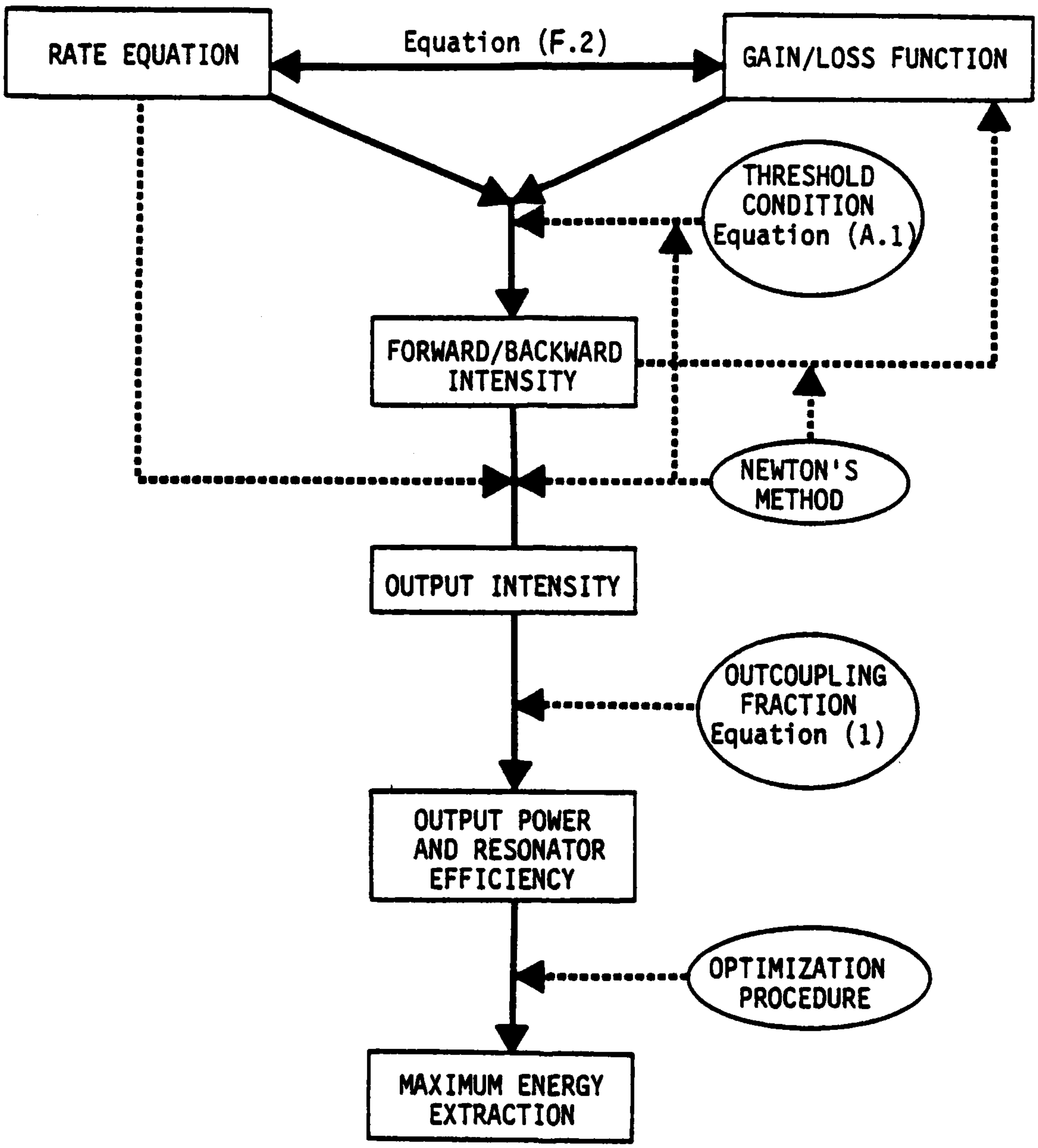


Figure 3 Flowchart of the procedure for evaluation of the maximum energy extraction.

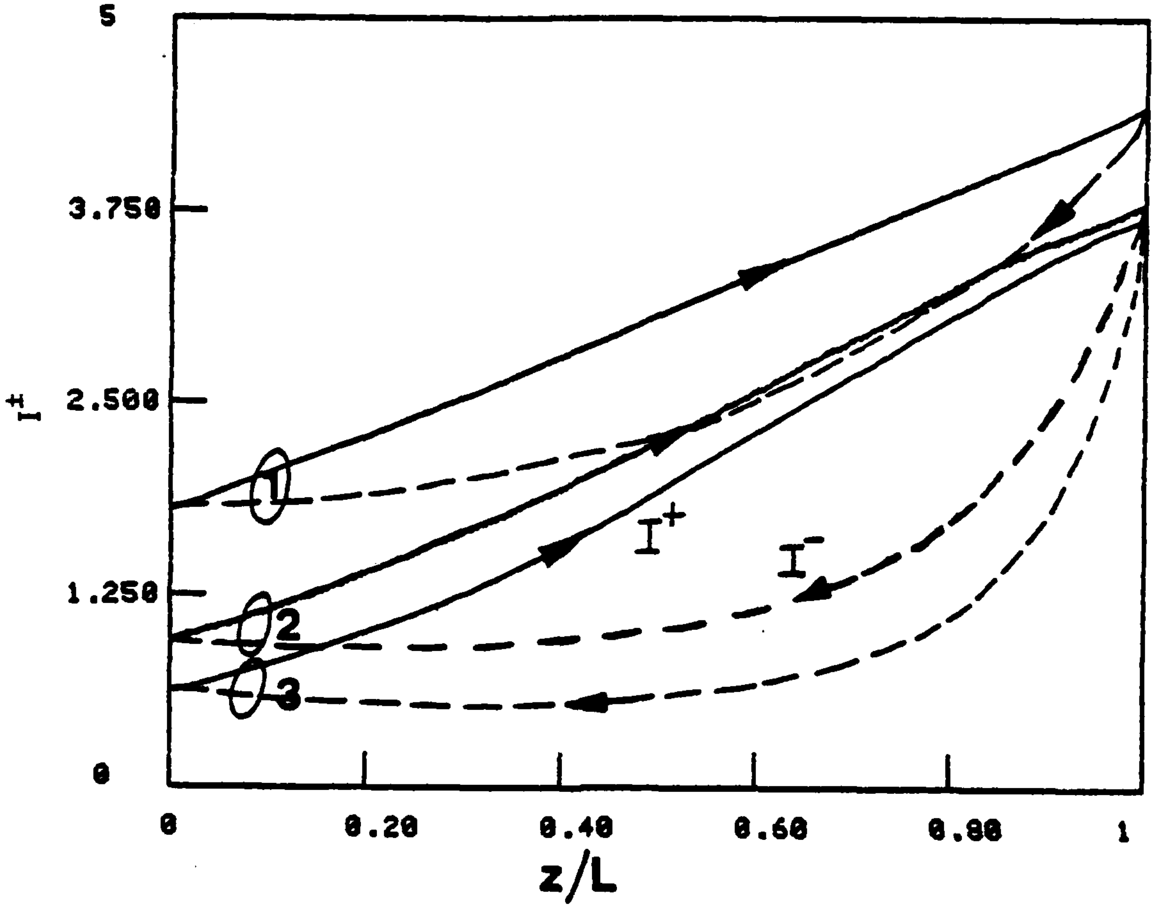


Figure 4 (A). The z-dependence of the forward (I^+) and backward (I^-) intensity inside the confocal resonator with magnification M = (1) 2, (2) 4 and (3) 6. System parameters are: $g_0L = 6$, $\alpha = 2 \times 10^{-3}$ cm^{-1}, L = 35 cm, a = 1.5 cm and $R_1 = R_2 = 1$.

Figure 4.b. It is seen that stronger r-dependence is associated with smaller magnification and the intensity profiles are almost linearly dependent on the transverse coordinate, r. This feature allows us to evaluate the output power, Equation [1], in a simpler way. For spherical (2D) mirrors, we obtain

$$P = \pi a^2(1 - M^{-2})\, C_1 + \pi a^3(1 - M^{-3})\, C_2/3, \qquad [8]$$

for an output intensity given by $I_{out} = C_1 + C_2 r$. It is interesting to see that the above expression reduces to the conventional form with a constant outcoupling fraction $(1 - M^{-2})$ when B = 0. We, however, should also note that both A and B are M-dependent and the actual explicit form of P in terms of M are not analytically available.

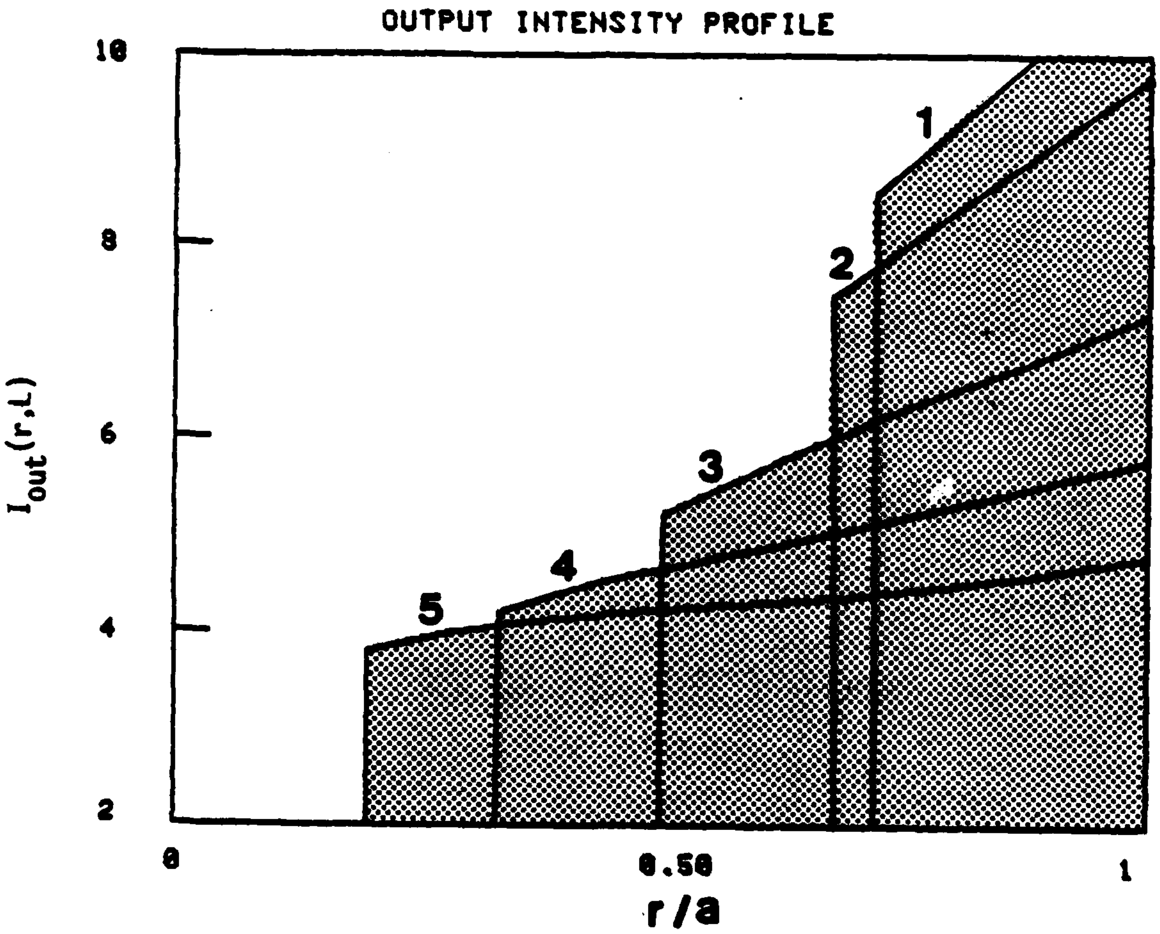


Figure 4 (B). The output intensity profiles $I_{out}(r, L)$ for a confocal unstable resonator with magnification M = (1) 1.4, (2) 1.5, (3) 2, (4) 3 and (5) 5.

V. DISCUSSION AND CONCLUSION

We summarize the numerical results in the following discussions:

(i) As shown in Figure 5, the efficiency based on the mean-field approximation (MFA), Equation [2], over-estimates that of the numerical (exact) results, particularly in the large M regime. Moreover, the maximum efficiency is also over-estimated by the expression of Equation [4]. We note that the optimal magnification based on the near-field information does not simultaneously meet the requirement of good beam quality, i.e., high focusability. To show this, we also plot the M-dependence of the center intensity of the far-field in Figure 5. We see that high efficiency at lower M and high beam quality at higher M are two competing factors which should be optimized.

(ii) The effects of the gain and loss ration, $D = g_0L/\alpha_0L$, on the efficiency profiles are shown in Figure 6. It is easy to see that higher D values result in higher efficiency and the optimal magnification is shifted to lower values when D increases. These features based on the numerical results may be clearly seen from the approximate expressions Equations [4] and [5].

(iii) In Figure 7, we show the effects of the unsaturated gain for fixed values of D. It is seen that the numerically calculated peak efficiencies, depending on the gain, deviate slightly from those of the approximate expression, Equation [4], in which η_0^* should be independent of the gain as far as the D value is fixed. Moreover, the general feature of the optimal magnification which increases as the gain increases is well described by Equation [5].

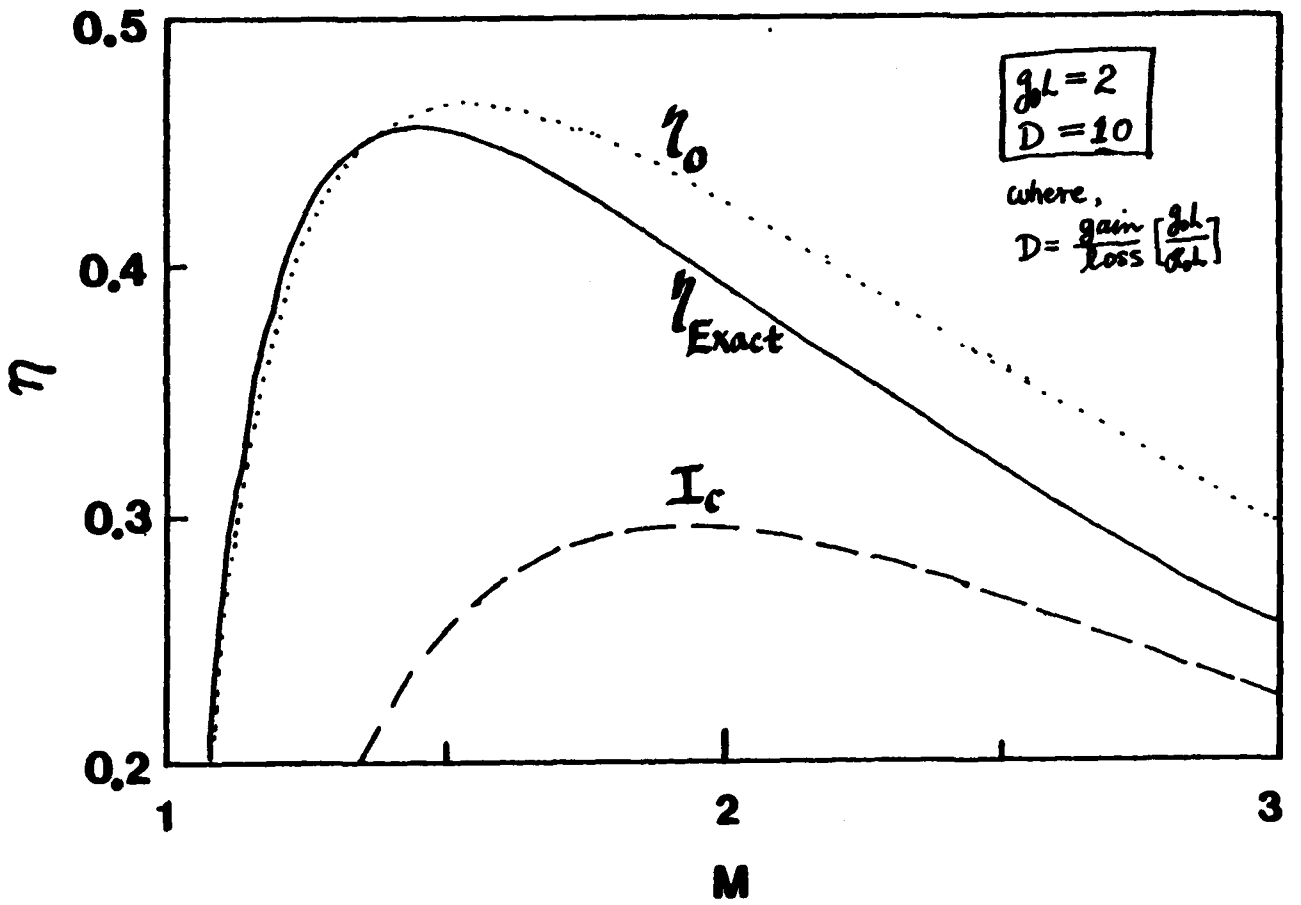


Figure 5. *Energy extraction efficiency versus magnification obtained numerically (solid curve) and approximately (dotted curve, based on Equation [2]). Also shown is the M-dependence of the center intensity of the far-field (see Appendix D).*

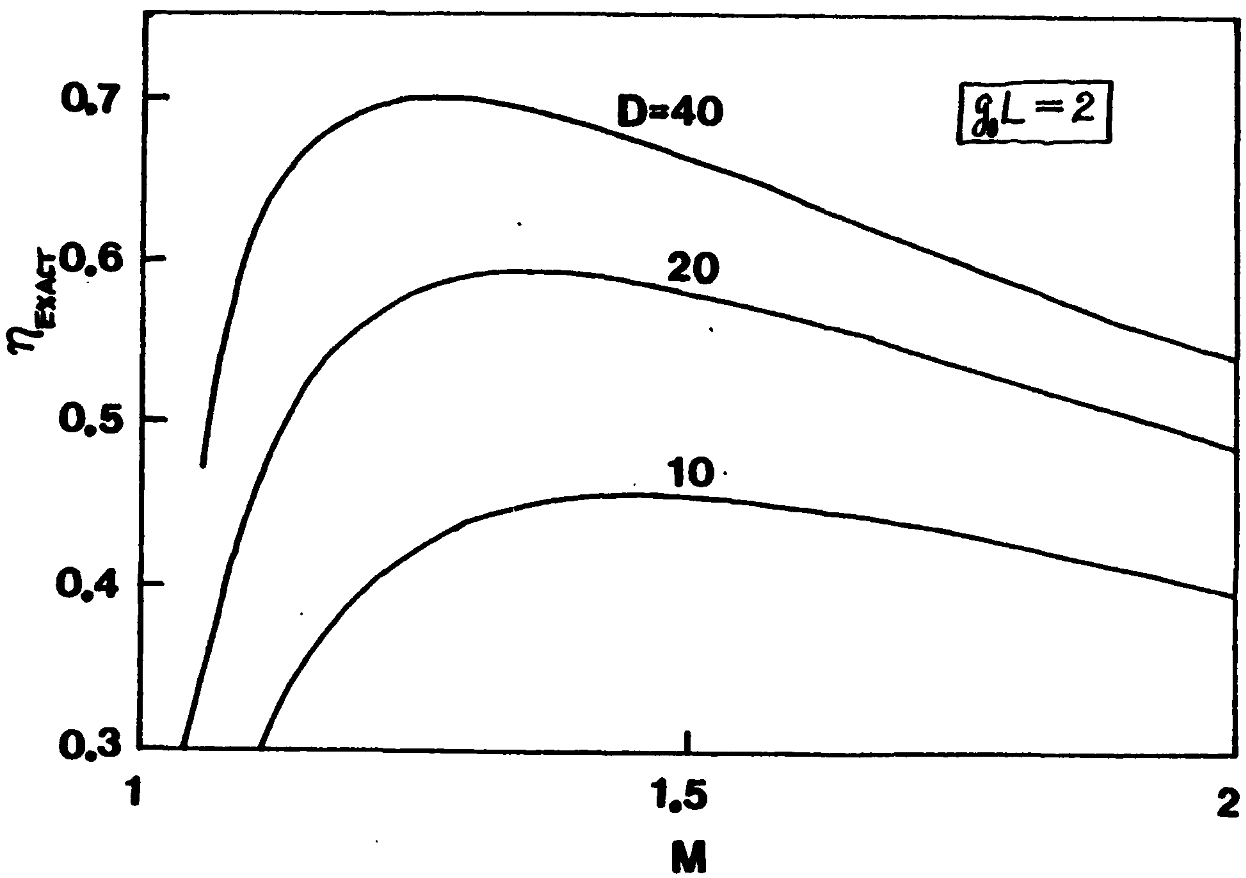


Figure 6. Effects of the gain and loss ratio (D) on the efficiency profiles for a fixed value of $g_0L = 2$.

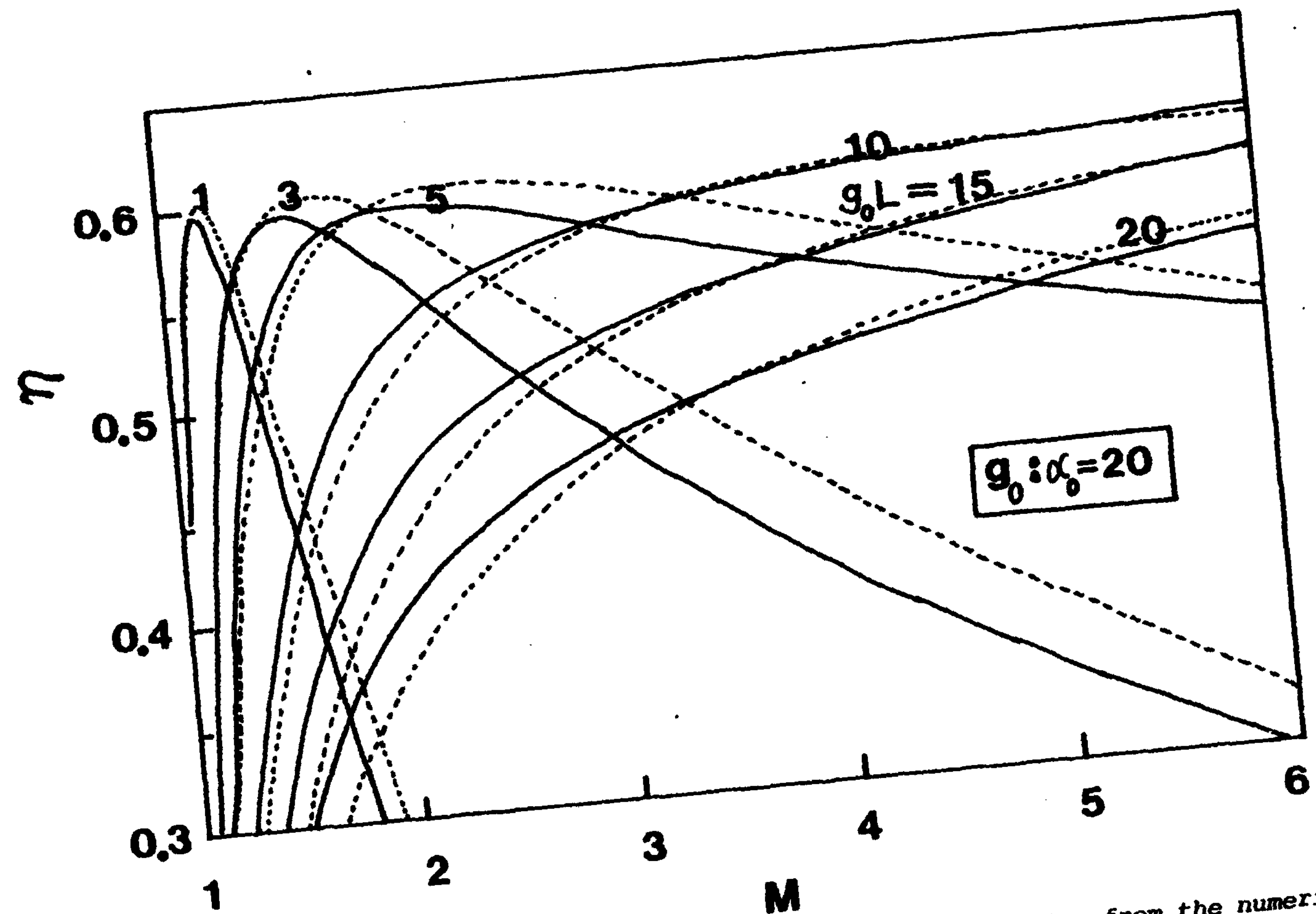


Figure 7. The effects of the unsaturated gain on the efficiency profiles from the numerical (solid) and approximate (dotted) results.

(iv) In Figure 8, we show the center intensity of the far-field which, in contrast to the (near-field) efficiency, is a strongly increasing function of the gain, for a fixed D value.

(v) To provide some guidance of the optimal design, we plot the gain and magnification diagram in Figure 9. For a given magnification, there are two values of the unsaturated gain which result in the same efficiency, say 55 percent. However, the lower one, g_1, is preferred as far as pumping is concerned. We note that the maximum efficiency, η^*, may not always be achieved and is restricted by the values of M, which is finite. For example, when $g_0L = 10$, efficiency of 55 percent is the most one may achieve. To achieve a higher efficiency, one may "tune" the gain to meet the optimal condition for a fixed magnification or vice versa.

(vi) For the situation that some of the system parameters are not well-defined or may not be measured accurately, the analytic expressions of the output power and the related optimal condition are very useful for the analysis of unknown parameters. For example, the measurements of the output power at the low M and high M regimes together with the beam size will provide us the information of the maximum output power and the corresponding optimal magnification. For greater detail, see Appendix D.

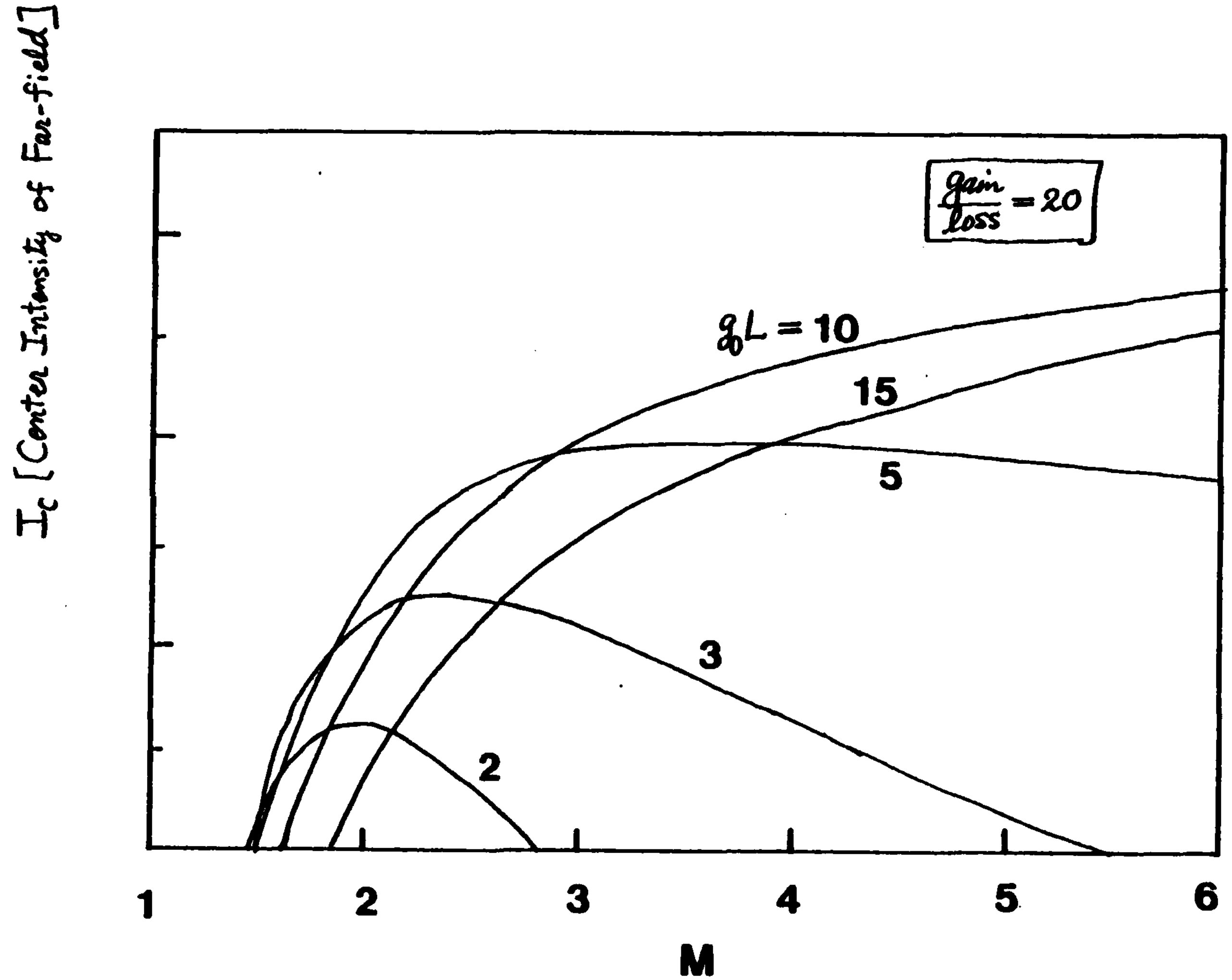


Figure 8. Effects of the unsaturated gain on the center intensity of the far-field in which an averaged intensity illuminates on the output spherical mirror with aperture size 2 a/M.

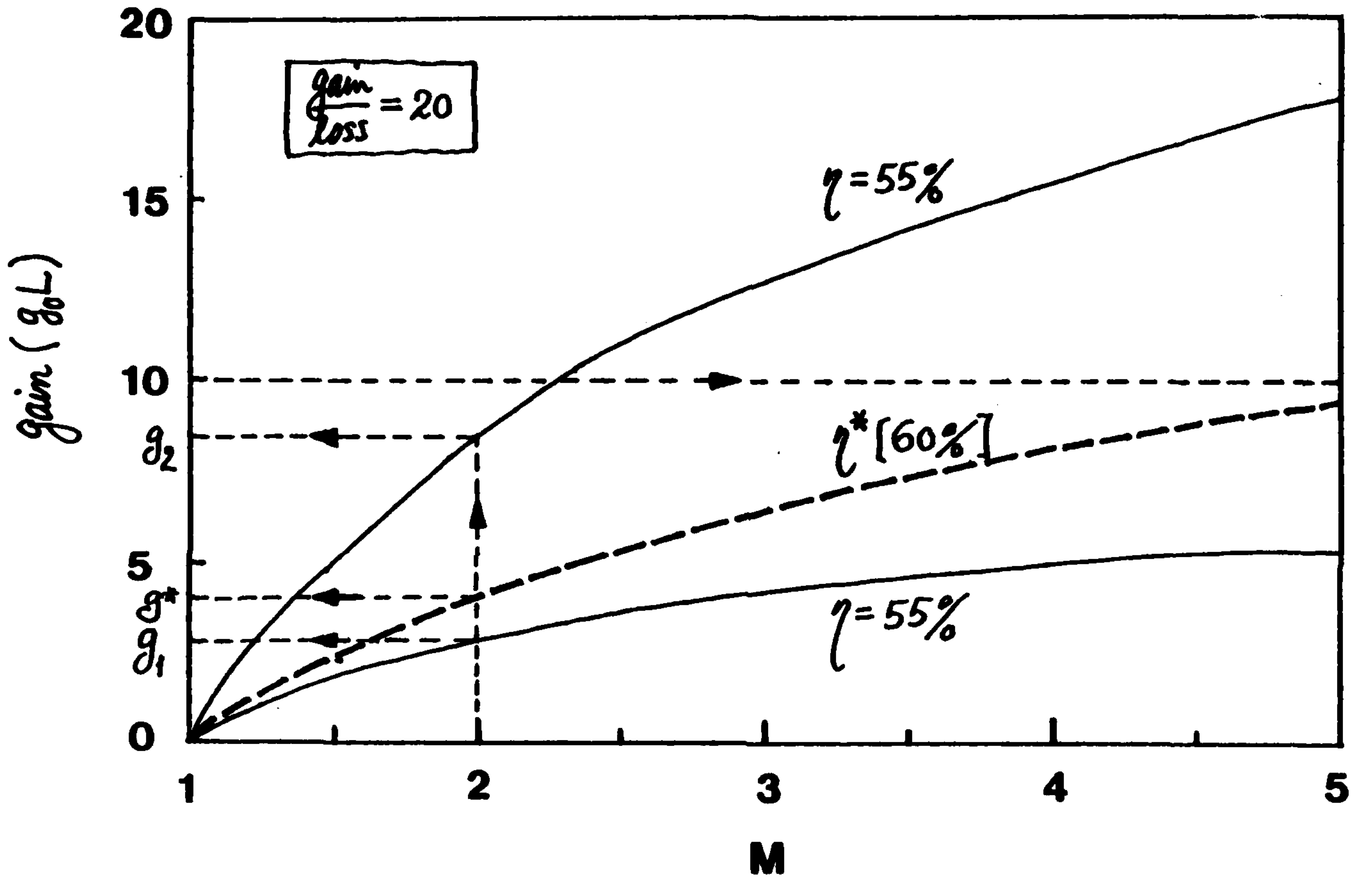


Figure 9. Unsaturated gain versus magnification associated with Figure 8 at the efficiency of 55 percent and at the maximal efficiency [60 percent].

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APPENDIX A

Efficiency in the Mean-Field Approximation (MFA): A Generalized Rigrod Calculation

As mentioned previously, most of the Rigrod-type calculations for the energy extraction efficiency are based on the constant intensity product $I_+ \bullet I_-$ = constant, which is true only for planar resonators. For unstable resonators with curved mirrors, both $I_+ \bullet I_-$ and $(I_+ + I_-)$ are strongly *z-dependent* as shown in our numerical results. In the following discussion we shall solve the rate equation for a telescopic resonator, positive branch, confocal unstable resonator, within a mean-field approximation(MFA). Analytic expression for the energy extraction efficiency will be derived and the limiting case with M=1 (planar resonator) will be compared with that of the Rigrod's results.

For a resonator with uniform gain distribution in the transverse direction, the 2-D rate equation is reduced to the following 1-D case

$$\frac{1}{I_+}\frac{dI_+}{dz} = g-\alpha_0 \, , \tag{A.1}$$

$$\frac{1}{I_-}\frac{dI_-}{dz} = -(g-\alpha_0)+\frac{2}{z_0-z} \, , \tag{A.2}$$

$$z_0 = \frac{ML}{M-1} \, , \tag{A.3}$$

which has the formal solution

$$I_+(z) = I_1 \cdot e^{\int_0^z [g-\alpha_0]dz'} \tag{A.4}$$

$$I_-(z) = I_4 \left(\frac{z_0}{z_0-z}\right)^2 \cdot e^{-\int_0^z [g-\alpha_0]dz'} \, , \tag{A.5}$$

where the pre-exponential z-dependence factor is due to the beam expansion of the backward field in a 2-D spherical-mirror system.

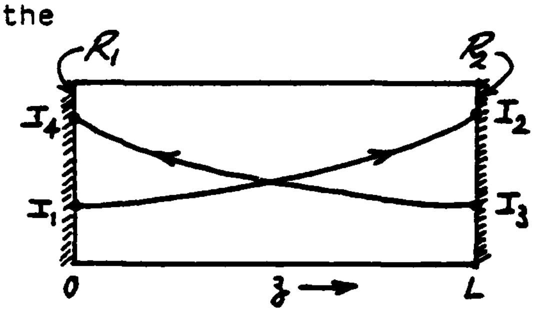
We readily to see that, from (A.4), (A.5), the intensity product

$$I_+(z) \bullet I_-(z) = I_1 I_4 \left[\frac{z_0}{z_0-z}\right]^2 = C_z \, , \tag{A.6}$$

is z-dependent for M>1, and is a constant for the planar case, M=1. The associated gain function, for a homogeneous broadening, is given by

$$g(z) = \frac{g_0}{1+I_++I_-} = \frac{g_0}{1+I_++C_z/I_+} \, . \tag{A.7}$$

We shall now solve the rate equation subjected to the boundary conditions on the mirrors with reflectivities R_1 and R_2:



$$
\begin{aligned}
&I_1 = R_1 I_4 \,, \\
\text{(A.8)} \qquad &I_3 = R_2 I_2 \,, \\
&I_4/I_2 = \sqrt{\frac{R_2}{R_1}}\Big/ M \,, \\
&I_2/I_1 = M\Big/\sqrt{R_1 R_2} \,.
\end{aligned}
$$

There are several ways to solve the rate equation, however, in order to employ the MFA for the total intensity, we substitute (A.7) into (A.1) and rearrange it in an integral form

$$\int_{I_1}^{I_2} \left[1 + \frac{1}{I_+} + \frac{C_z}{I_+^2} \right] dI_+ = \int_0^L \left[(g_0 - \alpha_0) - \alpha_0 (I_+ + I_-) \right] dz \,. \qquad \text{(A.9)}$$

In general, the above equation is not in a s parated form of z and I_+ due to the z-dependent C_z and the I_+-dependent total intensity. For analytic form of these integrals, we shall impose the MFA for the total intensity in the RHS of (A.9), and use the zeroth-order of the expansion of C_z in the LHS of (A.9). Since $z/Z_0 < 1$, we may expand

$$C_z \cong I_1 I_4 \left[1 + \frac{2z}{Z_0} + 3\left(\frac{z}{Z_0}\right)^2 + \cdots \right]. \qquad \text{(A.10)}$$

Within the MFA,

$$I_+(z) \approx I_1 e^{\bar{K} z} = I_2 e^{\bar{K}(z-L)} , \qquad \text{(A.11)}$$

$$I_-(z) \approx I_4 \left(\frac{Z_0}{Z_0 - z} \right)^2 e^{-\bar{K} z} \approx \sqrt{\frac{R_2}{R_1 M^2}}\, I_2 \left[1 + \frac{2z}{Z_0} + \cdots \right] e^{-\bar{K} z} , \qquad \text{(A.12)}$$

where we have defined the mean value

$$\bar{K} = \frac{1}{L} \int_0^L [g - \alpha]\, dz \approx \bar{g} - \alpha_0 = \frac{g_0}{1 + \overline{I_+ + I_-}} - \alpha_0$$

and we have used the boundary conditions in (A.8).

Using (A.11) and (A.12), we may calculate the total intensity in the MFA:

$$\overline{I_+ + I_-} = \frac{1}{L}\int_0^L [I_+(z) + I_-(z)]\,dz = \frac{I_2}{\bar{K}L}\left[\left(1+\sqrt{\frac{R_2}{R_1M^2}}+\frac{2\epsilon}{\bar{K}L}\right)\left(1-e^{-\bar{K}L}\right) - 2\epsilon\sqrt{\frac{R_2}{R_1M^2}}\,e^{-\bar{K}L}\right], \quad \text{(A.13)}$$

with $\epsilon = 1 - M^{-1}$,

which may be re-written as, by using the steady-state(on axis) condition

$$R_1R_2M^{-2}\cdot e^{2\bar{K}L} = 1, \quad \text{(A.14)}$$

$$\overline{I_+ + I_-} = \frac{I_2}{\ln\sqrt{\frac{M^2}{R_1R_2}}}\left[\left(1-\sqrt{\frac{R_1R_2}{M^2}}\right)\left(1+\sqrt{\frac{R_2}{R_1M^2}}\right) + 2\epsilon\left(\sqrt{\frac{R_1R_2}{M^2}}\left(\ln\sqrt{\frac{R_1R_2}{M^2}} - 1\right) - \ln\sqrt{\frac{R_1R_2}{M^2}}\right)\right]. \quad \text{(A.15)}$$

Within the MFA,RHS of (A.9) is given by

$$RHS = (g_0 - \alpha_0)L - \alpha_0 L\,\overline{(I_+ + I_-)}. \quad \text{(A.16)}$$

We shall now evaluate LHS of (A.9). Using the leading term of (A.10), we are able to work out this integral follows:

$$\begin{aligned} LHS &\approx \ln\left(\frac{I_2}{I_1}\right) + (I_2 - I_1) + I_4\left(1 - \frac{I_3}{I_2}\right) \\ &= \ln\sqrt{\frac{M^2}{R_1R_2}} + I_2\left(1-\sqrt{\frac{R_1R_2}{M^2}}\right) + \sqrt{\frac{R_2}{R_1M^2}}\,I_2\left(1-\sqrt{\frac{R_1R_2}{M^2}}\right) \end{aligned} \quad \text{(A.17)}$$

From LHS=RHS, we solve for the intensity on the output mirror, I_2,

$$I_2 = \left(\frac{X}{f_1}\right)\left[\frac{g_0L}{\alpha_0L + X} - 1\right]\left[1 - 2\epsilon\left(\frac{\alpha_0 L f_2}{f_1}\right)(\alpha_0 L + X)\right], \quad \text{(A.18)}$$

where

$$X = \frac{1}{2}\ln\left(\frac{M^2}{R_1R_2}\right), \quad \text{(A.19)}$$

$$f_1 = \left(1+\sqrt{\frac{R_2}{R_1M^2}}\right)\left(1-\sqrt{\frac{R_1R_2}{M^2}}\right), \quad \text{(A.20)}$$

$$f_2 = \sqrt{\frac{R_1R_2}{M^2}}(X+1) - X. \quad \text{(A.21)}$$

$$\epsilon = 1 - M^{-1}.$$

WE note that the above expression reduces to that of Rigrod for the case of planar mirrors where M=1, and hence $X = -\ln(R_1R_2)$, with $\epsilon = 0$. We also note that the above new expression for the output intensity of a telescopic resonator shows its M-dependence through X, f_1, f_2 and ϵ. These results have not been derived in the literature, although the high magnification case with $f_1 = 1$, and $\epsilon = 0$, has been used in Ref. 2,3.

Knowing the output intensity acrossing the output mirror, I_2, we are able to evaluate the output power , from Eq.(1),

$$P_{out} = 2\pi I_S \cdot \int_{a/M}^{a} I_2 \cdot r\,dr \approx \pi I_S a^2 (1-M^{-2}) \bar{I}_2 \,, \tag{A.22}$$

where we have used a mean value such that a constant outcoupling fraction $(1-M^{-2})$ is defined in the above equation. In general I_2 is transverse coordinate dependent and (A.22) needs numerical method. Substituting (A.18) into (A.22), we may calculate the efficiency within the MFA and constant outcoupling approximation as follows.

$$\eta = \frac{P_{out}}{\pi a^2 g_0 L} \approx \left(\frac{1-M^{-2}}{f_1}\right)\left(\frac{X}{g_0 L}\right)\left[\frac{g_0 L}{\alpha_0 L + X} - 1\right](1-2\epsilon F),$$

$$F \approx \left(\frac{\alpha_0 L f_2}{f_1}\right)(\alpha_0 L + X) . \tag{A.23}$$

Note that $2\epsilon F$ is the first-order correction term resulted from the expansion of C_z, Eq.(A.10). In the zeroth-order approximation and when $R_1 = R_2 = 1$, Eq.(A.23) reduces to Eq.(2). Furthermore, as shown by our numerical results that the output intensity is only slightly depending on the transverse coordinate for large M and a constant outcoupling $(1-M^{-2})$ is valid in Eq.(A.22). The overestimation of Eq.(3) may be caused by its neglecting the correction term.

We finally note that the MFA expression for the efficiency, (A.23), (2), fits well with the numerical results even for systems with high gain and loss ratio, within 5 % deviation. This is in contrast to that of the Rigrod's results where the MFA expression deviates from the numerical results significantly when the gain/loss is large.

APPENDIX B

Derivation of the Explicit Form of the Intensity Profiles without Using MFA

We shall now solve the rate equation, (A.1) and (A.2) subjected to the steady-state condition. We first should point out that the direct numerical solving the rate equation by, e.g., Runge-Kutta method is difficult due to: (i)the boundary conditions in the regime I and II and (ii)the intensity dependent gain function. In order to solve the rate equation by Newton's method, the explicit form of the gain function must be known. For this purpose, the total intensity is imposed as

$$I_+(z)+I_-(z) \approx 2\sqrt{I_+(z)I_-(z)} \approx 2I_0\left[\frac{z_0}{z_0-z}\right], \tag{B.1}$$

which is numerically justified and shown to be valid for $M<5$.

Using (B.1), the gain function is given by

$$g(z) \approx \frac{g_0}{1+2I_0 z_0/(z_0-z)}, \tag{B.2}$$

which is substituted into the steady-state condition and gives

$$I_0 = \left(\frac{M-1}{2M}\right)\left[(g_0-\alpha_0)L+\ln\sqrt{\frac{R_1R_2}{M^2}}\right] \Big/ \left[g_0 L \ln\left(1+\frac{M-1}{2I_0M+1}\right)\right], \tag{B.3}$$

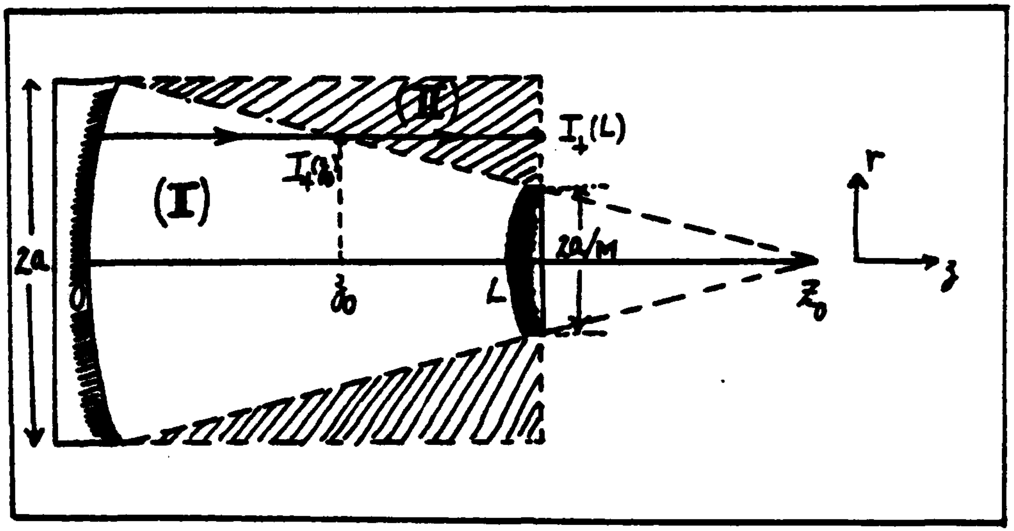
where $I_0=\sqrt{I_1 I_4}$ may be sovled by Newton's method from the above self-consistent equation.

Given the explicit form of the gain, we are able to work out the forward and backward intensities, from (A.4) and (A.5),

$$I_+(z) = \sqrt{R_1}\, I_0 \left[1-\frac{z/z_0}{2I_0+1}\right]^{2I_0 z_0 g_0} \cdot e^{(g_0-\alpha_0)z}. \tag{B.4}$$

$$I_-(z) = \left(\frac{z_0}{z_0-z}\right)^2 I_0 \Big/ I_+(z). \tag{B.5}$$

Above expressions give the on-axis intensity profiles. For intensity off-axis, the propagation of the forward field must be treated more carefully in the regime where I_- is absent.



As shown in the above figure, in the pure amplification regime(II), the gain is given by

$$g(z) = \frac{g_0}{1 + I_+(z)} \; , \tag{B.6}$$

and solution of the rate equation, (A.1), becomes

$$\int_{I_+(z_0)}^{I_+(L)} \frac{dI_+}{(g-\alpha_0)I_+} = \int_{z_0}^{L} dz \; , \tag{B.7}$$

where z_0 is r-dependent and is given by

$$z_0 = L - z_0\left(\frac{r}{a} - \frac{1}{M}\right) . \tag{B.8}$$

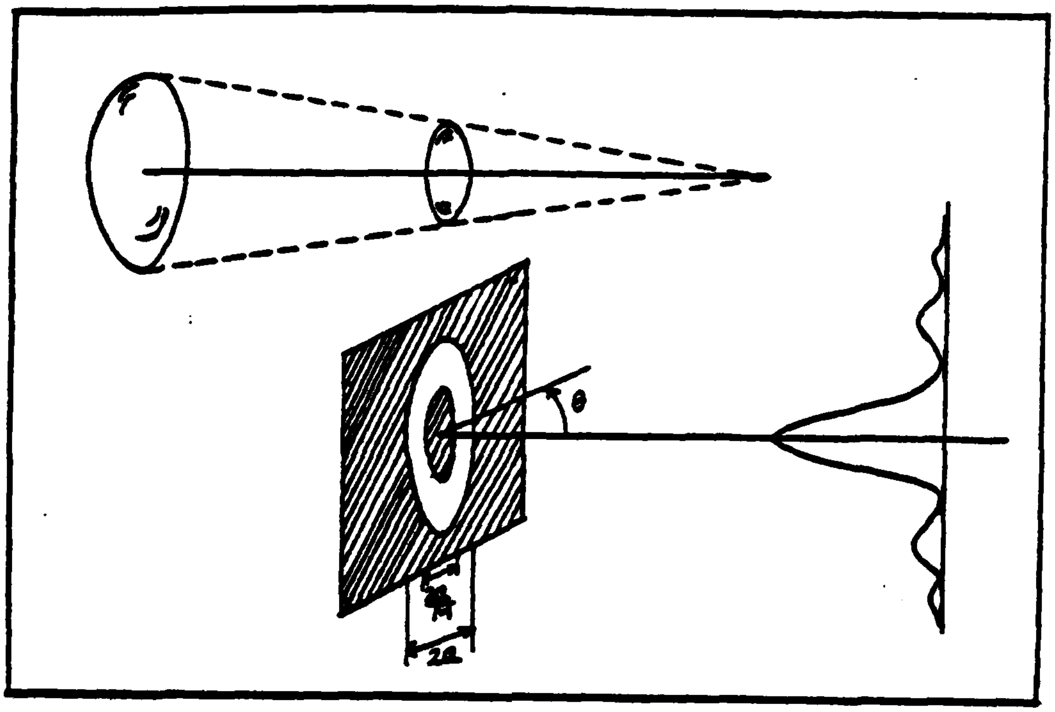
(B.7) may be easily worked out and a self-consistent equation is obtained

$$\ln\left[\frac{I_+(L)}{I_+(z_0)}\right] = (g_0 - \alpha_0)(L - z_0) + \left(\frac{g_0}{\alpha_0}\right) \ln\left[\frac{(g_0 - \alpha_0) - \alpha_0 I_+(L)}{(g_0 - \alpha_0) - \alpha_0 I_+(z_0)}\right] \tag{B.9}$$

Again, by employing Newton's method, we are able to solve the output intensity acrossing the output mirror plane for $a/M \leqslant r \leqslant a$. The input intensity in (B.9) is given by (B.4) with $z = z_0$ which is now r-dependent.

APPENDIX C

Far-Field Intensity for Unstable Resonators with Spherical Mirrors with the Geometric-Optics Limit



As show in the above figure, the far-field amplitute is given by[8]

$$U = C\left[\int_{-a}^{a} e^{iky\sin\theta}\sqrt{a^2-y^2}\,dy - \int_{-a/M}^{a/M} e^{iky\sin\theta}\sqrt{(a/M)^2-y^2}\,dy\right] \qquad \text{(C.1)}$$

$$= C\pi a^2\left[\frac{2J_1(\rho)}{\rho} - M^{-2}\frac{2J_1(\rho/M)}{\rho/M}\right], \quad \rho = ka\sin\theta,$$

where J_1 is the first-order Bessel function. Noting that $[J_1(\rho)/\rho]\rightarrow 1/2$ as $\rho\rightarrow 0$, the center intensity of the far-field is given by

$$I_c = |U(\rho=0)|^2 = I_\infty(1-M^{-2})^2, \qquad \text{(C.2)}$$

where I_∞ is the value for $M=\infty$ and we have used the mean output intensity incident upon the aperture

$$I_\infty = \frac{1}{2}\left[I_{out}(a/M) + I_{out}(a)\right]. \qquad \text{(C.3)}$$

We note that I_{out} depends on M and the outcoupling fraction $(1-M^{-2})$ is an increasing function of M, therfore, their produt,

I_c, has a maximun value at the optimal magnification which is shifted to larger value compared with that of the near-field efficiency. For the purpose of maximun extraction energy as well as high focusability, we shall re-define an efficiency based on the far-field energy rather than the near-field output power as discussed previously.

The new efficiency based on the far-field intensity is defined as[9]

$$\eta = \frac{E_1}{\pi a^2 g_0 L} \quad , \tag{C.4}$$

where E_1 is the center-ring energy of the far-field given by

$$E_1 = \frac{1}{E_\infty} \int_0^{\rho_1} I(\rho) 2\pi\rho \, d\rho , \tag{C.5}$$

and $I(\rho)$ is the far-field intensity given by $|U|^2$ in Eq. (C.1). We note that in an unloaded resonator E_1 increases as M increases, however, for a loaded resonator the energy extraction is saturated by the gain/loss and competing with the outcoupling fration $(1-M^{-2})$. Therefore,to perform a resonator with high energy extraction efficiency as well as high focusability, we shall choose an optimal magnification which is larger for larger gain systems.

APPENDIX D

Analysis of the Unknown Parameters via the Measurements of Output Power and Beam Area

Due to the high cost of the mirror, particularly for large aperture systems, it is highly unlikely that we may change the system magnification continuously to obtain the optimal condition. Therefore it is highly desirable to estimate the optimal magnification by some of the measurable laser parameters. In the following discussion, we shall suggest some possible procedures of determining the unknown parameters by some of the measurements.

From Equation [3], we readily see that experimental measurements of the gain and loss shall enable us to evaluate the optimal magnification. However, when the measurements of the gain and loss are inconvenient or inaccurate, there is an alternative to evaluate them. To demonstrate this mathematically, we combine the expressions of Equations [3] and [4] to find the output power

$$P_{out} = AI_s\eta_0 g_0 L = AI_s X \left[\frac{g_0 L}{\alpha L + X} - 1\right]. \qquad [D.1]$$

As shown in Figure D, the output power has slopes m_1 and m_2 at the low X and high X, respectively, given by

$$m_1 = AI_s X_0/(\alpha_0 L), \qquad [D.2a]$$

$$m_2 = AI_s, \qquad [D.2b]$$

where $X_0 = (g_0 - \alpha_0)L$ is the value at the oscillation threshold, A and I_s are the beam area and the saturation intensity at which $g = g_0/2$. For Equation [D.2b], we see that

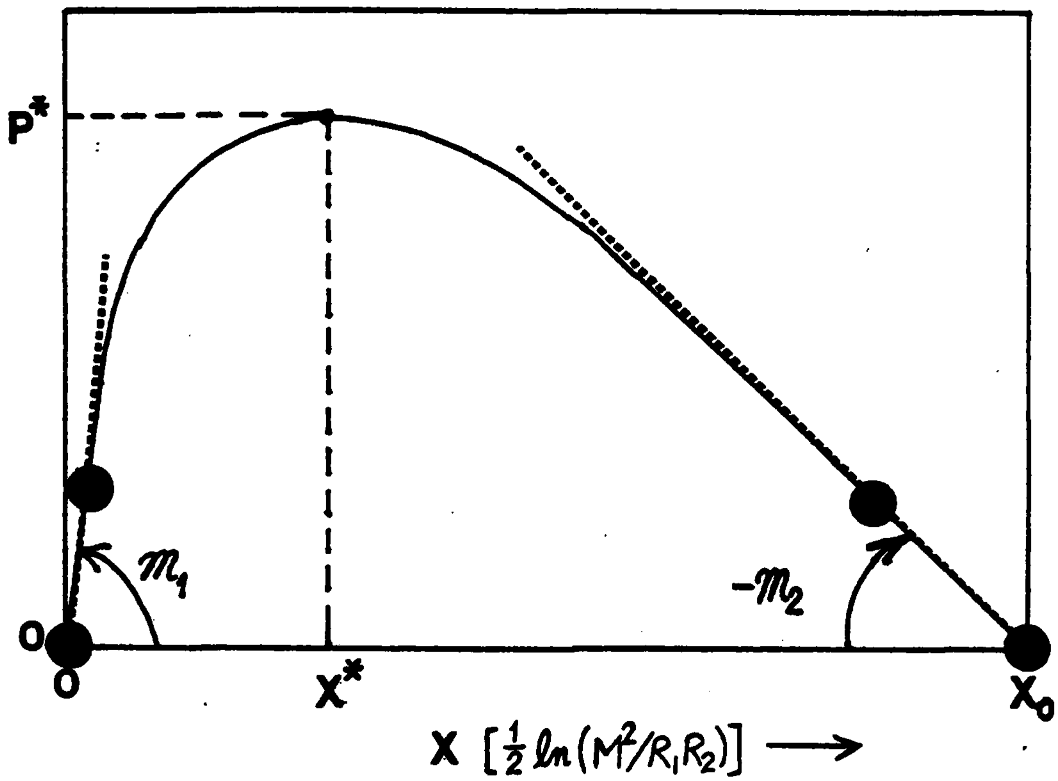


Figure D. Schematic diagram of the output power versus resonator parameter X. Note that measurements of the slopes M_1 and M_2 will provide us the optimal value X.*

measurement of A together with the slope m_2, defined by the threshold value X_0 and the measurement of the output power closes to that, enables us to determine I_s. From the measured slope m_1, X_0, A and the calculated I_s, we then may calculate the loss which, from the relation $X_0 = (g_0 - \alpha_0)L$, gives the gain. Knowing the gain and loss, we finally may evaluate the maximum efficiency and the corresponding optimal magnification from Equations [3] and [4].

APPENDIX E

Computer Code for the Energy Extraction Efficiency

```
10    C******** FIND THE OPTIMAL MAGNIFICATION  FOR UNSTABLE RESONATOR *******
11    C*******  EXTRACTION EFFICIENCY.  /RE.FOR/  J.T. LIN  2-10-84     *******
12    C******** IN THIS PROGRAM GAIN=GL(Z,0), WITHOUT R-DEPENDENCE **********
14    C******** RATE EQUATION IS 1-D , BUT ALLOW INTENSITY I=I(Y,R) **********
20    ********* EXACT(YE),  RIGROD(YR),  COATED SYS.(YC), CENTER FAR-FIELD(YF)
25    ************************************************************************
30              IMPLICIT REAL*4(A-H,O-W,Z)
40              DIMENSION YT(2),XM1(201),XM2(201),YC(201),YE(201),YR(201),YF(201)
50              LOGICAL ERROR
60              EXTERNAL FT,GT
70              COMMON  XN,GL,AL,XM,R1,R2,XL,A,R,YY
80              TYPE*,'ENTER GL,AL,XR,DN,NN= 5,0.25,1.,2.95,100'
90              READ(5,55) GL,AL,XR,DN,NN
100     55      FORMAT(4G,I)
110   C INPUT PARAMETERS: R1,2(REFLECT), XL(CAVITY LENGTH), A(OUTPUT MIRROR RADIU
120   C R(POSITION IN VERTICAL), XM(MAGNIF), (GL,AL)=(GAIN,LOSS)
130   C DIMENSION OF THE RESONATOR GIVEN BY XN  XN=1(2D) XN=2(3D)
135   C...... refelctivity of the output lenses at 193 nm wave (RX)
136             RX=0.04
140             R1=XR
150             R2=1.
160   c.......XL=cavity length only affects M* but not efficiency, here,
162   c.......we we reduce XL by a factor of 100. to avoid underflow....
170             XL=1.
180             A=1.5
190             XN=2.
200             DM=DN/FLOAT(NN)
201   C*******ABOVE RN GIVES THE HIGHEST RANGE OF XM, FROM 1 TO (RN+1)(3.95ETC)
210             NPTS=NN+1
220             DO 40 K=1,NPTS
230             XM=1.05 + FLOAT(K-1)*DM
240             DO 33 I=1,2
250             R=A*(FLOAT(I-1)*(XM-1.)+1.)/XM
260   c..... initial values for Newton method  x=1.  & 5. for FT & GT
270             X=1.0
290             CALL NEWTON(X,FT,ERROR)
300   C.......above result of X will be used as the input intensity in GT
310             YY=X
320             X=5.
330             CALL NEWTON(X,GT,ERROR)
340             YT(I)=X
350     33      CONTINUE
360             CM1=1. - 1./XM
370             CM2=1. - 1./XM**2
380             CM3=1.-1./XM**3
385   c.......ouput intensity approx. by linear fuct  betwwen yt(1) & yt(2)
386   C.......on-axis intensity (YIC)=YT(1)...........
387             YIC=YT(1)
390             CB=(YT(2)-YT(1))/(CM1*A)
400             CA=YT(2) - CB*A
410   C THE RESONATOR EFFICIENCES WILL BE NORMALIZED BY GL*A**XN FOR 1&2D(XN=1,2)
420             RIGROD=(GL/(AL +0.5*XN*ALOG(XM/SQRT(XR))) - 1.)
430             RD1=(GL/(AL +0.5*ALOG(XM)) - 1.)
432   c..... to compare exact(YE), Rigrod(YR),  coated(YC), far field int(YF)
440             YE(K)=(CA*CM2 + (2.*CB*A/3.)*CM3)/GL
450             YR(K)=0.5*XN*ALOG(XM/SQRT(XR))*RIGROD/GL
452   C..... find far-field center intensity, with mean int.(EA).....
```

```
      EA=CA+0.5*CB*A*(1.+1./XM)
      YF(K)=EA*CM2**2/GL
      XM1(K)=XM
   40 CONTINUE
      CALL PLOTPT(XM1,YE,NPTS)
      CALL PLOTPT(XM1,YR,NPTS)
      CALL PLOTPT(XM1,YF,NPTS)
      END
C*******   EXTERNAL FUNCTIONS ******************************
      FUNCTION FT(X,FX,DFX)
      COMMON  XN,GL,AL,XM,R1,R2,XL,A,R,YY
      C1=2.*X + 1.
      C2=2.*X*XM + 1.
      F1=ALOG(XM*C1/C2)
      F2=((XM-1.)/(2.*XM*GL))*(GL-AL-0.5*ALOG(XM**XN/(R1*R2)))
      FX=X - F2/F1
      DFX=1. - (F2/F1**2)*(XM-1.)/(C1*C2)
      RETURN
      END
C
      FUNCTION GT(X,FX,DFX)
      COMMON  XN,GL,AL,XM,R1,R2,XL,A,R,YY
C...... find intensity I(forward)  in regime where I(back)=0........
      Z=XM*XL/(XM-1.)
      EL=Z*(R/A - 1./XM)
      ZZ=XL-EL
      Y1=(1.-ZZ/(Z*(2.*YY+1.)))**(2.*YY*Z*GL/XL)
      Y=SQRT(R2)*YY*EXP((GL-AL)*ZZ/XL)*Y1
      G1=(GL-AL-AL*X)/(GL-AL-AL*Y)
      FX=ALOG(ABS(X/Y)) - (GL-AL)*(EL/XL) - (GL/AL)*ALOG(ABS(G1))
      DFX=1./X + GL/(G1*(GL-AL-AL*Y))
      RETURN
      END
C*****************************************************************
      SUBROUTINE NEWTON(X,FUNC,ERROR)
      COMMON  XN,GL,AL,XM,R1,R2,XL,A,R,YY
      LOGICAL ERROR
      ERROR=.FALSE.
      TOL=1.0E-4
      MAX=50
      DO 20 I=1,MAX
      X1=X
      CALL FUNC(X,FX,DFX)
      IF(DFX .EQ. 0.0) GOTO 99
      DX=FX/DFX
      X=X1 - DX
      IF(ABS(DX) .LE. ABS(X*TOL)) GOTO 30
   20 CONTINUE
      ERROR=.TRUE.
   30 RETURN
   99 ERROR=.TRUE.
      RETURN
      END
```